Regime Switching Consistent Expectations Equilibria: General Results with Application to Monetary Policy

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Introduction

- Rational Expectations (RE): realistic assumption?
  - Agents know everything about the true underlying model of the economy.
  - Have a lot of computational power to solve the model.
  - One needs to impose a lot of structural frictions to replicate certain data features.
- One way of departing from RE
  - Stochastic Consistent Expectations (SCE)

Introduction

- What are SCE?
  - Agents have no knowledge of the true underlying model of the economy.
  - They form expectations based on a *mis-specified* perceived law of motion (PLM).
  - When moments of the data match the perceived ones from the PLM, agents believe their PLM is correct.
  - Whenever there is a moments' match, *self-confirming* restricted perception equilibrium arises instead of the RE one.

This paper

- Lays out the framework of how to work with SCE in a N-regime switching univariate generic model (contribution).
  - Sufficient conditions under which at least a SCEE is guaranteed.
- Incorporates SCE in an exogenous regime switching model (contribution).
  - Fisherian equation combined with Taylor-type of monetary policy (MP).
  - MP switches aggressiveness following an exogenous Markov chain.

Literature Review

- Exogenous regime switching with fully rational agents
  - **Davig and Leeper ('07, AER)**
  - Farmer et. al. ('10, AER)
- Behavioral learning equilibria
  - **Hommes and Zhu ('14, JET)**
- Moments computations in Markov switching models
  - Bianchi ('16, JE)
  - Cho ('16, RED)

Generic Framework

Model

$$x_t = a(s_t) + b(s_t)\widetilde{E}_t x_{t+1} + c(s_t)y_t + u_t \tag{1}$$

$$y_t = \rho y_{t-1} + \varepsilon_t \tag{2}$$

$$\varepsilon_t \sim \text{iid}(\bar{\varepsilon}, \sigma_\varepsilon^2) \tag{3}$$

$$u_t \sim \text{iid}(\bar{u}, \sigma_u^2) \tag{4}$$

where

- $s_t \in \{1, 2, ..., N\}$ with exogenous transition prob. $p_{ij} = Prob(s_t = j \mid s_{t-1} = i)$
- Ergodic Markov-chain transition matrix

$$P = \begin{bmatrix} p_{11} & p_{12} & \dots & p_{1N} \\ p_{21} & p_{22} & \dots & p_{2N} \\ \dots & \dots & \dots & \dots \\ p_{N1} & p_{N2} & \dots & p_{NN} \end{bmatrix}$$

Generic Framework

Bounded Rationality, PLM

- Agents do not know the economy is driven by y_t and u_t
- They do not know there is a regime switch in the economy

Perceived Law of Motion (PLM)

$$x_t = \alpha + \beta(x_{t-1} - \alpha) + \delta_t \tag{5}$$

where

- $\alpha \in \mathbb{R}$
- $\beta \in (-1, 1)$
- $\delta_t \sim \text{WN}$

Forecast

$$\widetilde{E}_t x_{t+1} = \alpha + \beta^2(x_{t-1} - \alpha) \tag{6}$$

Generic Framework

Bounded Rationality, ALM

Actual Law of Motion (ALM):

$$x_t = k_i + l_i x_{t-1} + m_i y_t + u_t \tag{7}$$

where

- $k_i = a(s_t = i) + \alpha(1 - \beta^2)b(s_t = i)$
- $l_i = \beta^2 b(s_t = i)$
- $m_i = c(s_t = i)$

for any $i \in \{1, 2, ..., N\}$.

Generic Framework

SCEE Definition

Definition. (α, β) where $\alpha, \beta \in \mathbb{R}$ and $\beta \in (-1, 1)$, is called a first-order SCEE if the following three conditions are satisfied:

- The ALM has unconditional mean α, i.e., $\mathbb{E}(x_t \mid \text{ALM}) = \alpha$
- The ALM has unconditional first-order autocorrelation coefficient $\beta \in (-1, 1)$, i.e., $\text{Corr}(x_t, x_{t-1} \mid \text{ALM}) = \beta$

Application: Model

$$R_t = \widetilde{E}_t \pi_{t+1} + r_t \tag{8}$$

$$r_t = \rho r_{t-1} + \varepsilon_t \tag{9}$$

$$R_t = \phi_i (\pi_t - \pi_t^*) \tag{10}$$

$$\pi_t = \frac{1}{\phi_i} \widetilde{E}_t \pi_{t+1} + \frac{1}{\phi_i} r_t + \pi_t^* \tag{11}$$

- ϕ_i – MP aggressiveness in regime i, $i \in \{1, 2\}$.
- Regimes switch according to an exo Markov chain with transition prob.
  - $p_{ij} = P[s_t = j \mid s_{t-1} = i]$
- $\pi_t^* \sim \mathrm{N}\ (0, \sigma_\pi^2)$
- $\varepsilon_t \sim \mathrm{N}\ (0, \sigma_\varepsilon^2)$

Actual Law of Motion (ALM)

Using the generic framework, the model implies the following state-dependent ALM:

$$\pi_t = \frac{\delta(1-\gamma^2)}{\phi_i} + \frac{\gamma^2}{\phi_i}\pi_{t-1} + \frac{1}{\phi_i} r_t + \pi_t^* \tag{12}$$

Guaranteed Existence

Proposition 1. Given $\phi_i > \sqrt{p_{ii}}$, for any $i \in \{1, 2\}$, a **sufficient** condition for the existence of at least one SCEE (α^*, β^*) is

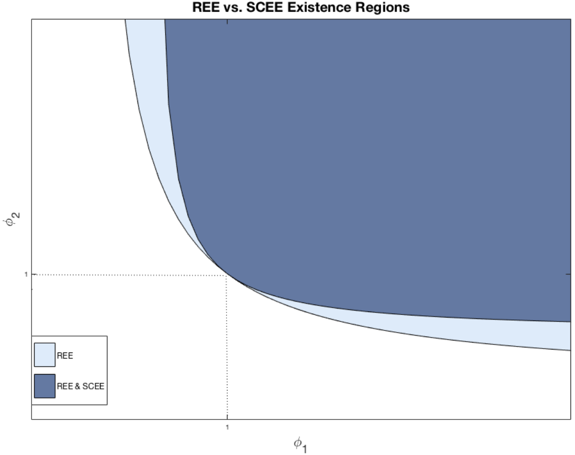
$$(1 - \phi_2^2)p_{11} + (1 - \phi_1^2)p_{22} + \phi_1^2\phi_2^2 > 1 \tag{13}$$

Remark. Davig and Leeper (AER, '07) show that, given $\phi_i > p_{ii}$ for any $i \in \{1, 2\}$, a unique bounded REE exists if

$$(1 - \phi_2)p_{11} + (1 - \phi_1)p_{22} + \phi_1\phi_2 > 1 \tag{14}$$

i.e., if the Long-Run Taylor Principle (LRTP) holds

Guaranteed Existence



Relative Moments

Relative Moment $= \text{Moment}_{\text{SCE}}/\text{Moment}_{\text{RE}}$

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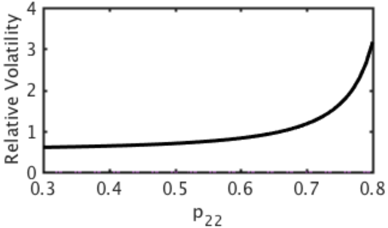
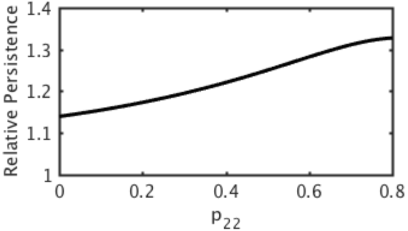
- **Mean**: $\alpha^* = 0$ ($=$ mean$_{\text{RE}}$)

Relative Moments

Relative Moment $= \text{Moment}_{\text{SCE}}/\text{Moment}_{\text{RE}}$

- **Mean**: $\alpha^* = 0$ $(= \text{mean}_{\text{RE}})$
- **Relative Volatility & Persistence**:

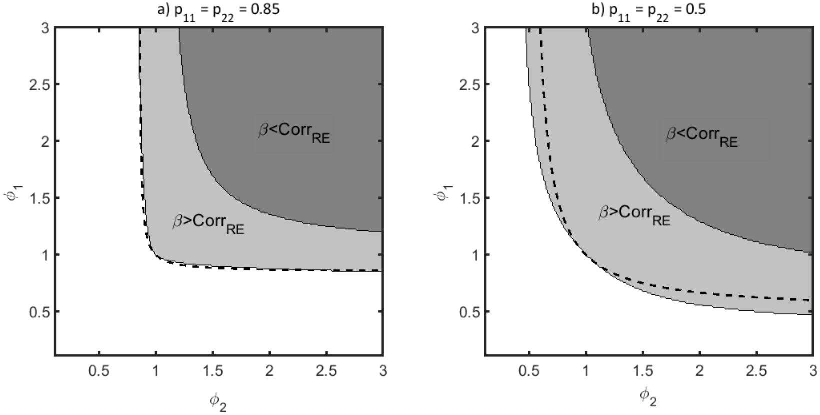




Parametrization: $\phi_1 = 1.5$, $\phi_2 = 0.9$, $p_{11} = 0.85$, $\rho = 0.8$, $\frac{\sigma^2_{\pi^*}}{\sigma^2_{\varepsilon}} = 1$

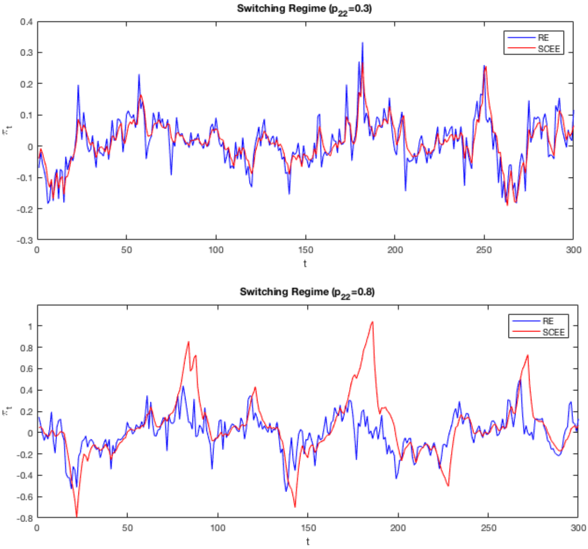
Existence

Calibration: $\rho = 0.8$, $\frac{\sigma^2_{\pi^*}}{\sigma^2_{\varepsilon}} = 1$



Multiplicity

Inflation Dynamics



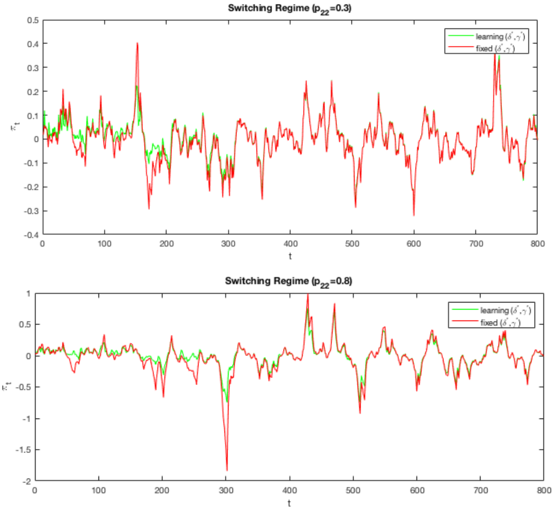
Sample Autocorrelation Learning (SAC)

- $\alpha_t = \frac{1}{t+1}\sum_{j=0}^{t}\pi_j$
- $\beta_t = \frac{\sum_{j=0}^{t-1}(\pi_j-\alpha_t)(\pi_{j-1}-\alpha_t)}{\sum_{i=0}^{t}(\pi_i-\alpha_t)^2}$

Proposition 2. A SCEE equilibrium $(0, \beta^*)$ is stable under SAC-learning if

$$\frac{\partial \mathbb{E}(\pi_t)}{\partial \alpha}\,\Big|_{(0,\beta^*)} < 1 \text{ and } \frac{\partial \mathrm{Corr}(\pi_t, \pi_{t-1})}{\partial \beta}\,\Big|_{(0,\beta^*)} < 1$$

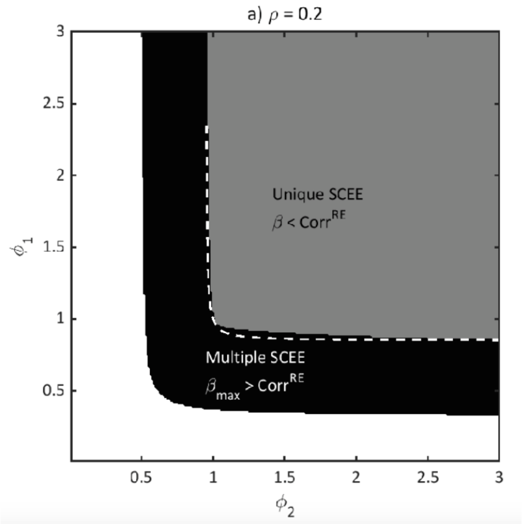
Sample Autocorrelation Learning (SAC)



Concluding Remarks

- The LRTP needed for a determinate REE does not eliminate fluctuations driven by bounded rationality.
- A wide variety of policy combinations induces equilibria with excess inflation persistence.
- The combination of an active with a persistently passive enough rule induces excess inflation volatility.
- Convergence to the equilibrium happens faster the less the passive regime lasts.

Multiplicity of SCEE



Existence